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Topic for

Semester – 3, Paper – PHSA CC6

PRESSURE EXERTED BY GAS

Assumptions of Kinetic Theory

(a) Gas molecules are continuously colliding against one another. Still in the steady state these collisions do not affect the molecular density in any element of volume of the gas.
The molecules do not gather at any place in larger numbers than at others.

(b) Between two successive collisions a molecule moves along a straight path with a uniform velocity. This straight path is called free path.

(c) The dimension of a molecule can be neglected in comparison with the distance travelled by it in between two collisions.

(d) The time during which a collision lasts is negligible compare to the time taken by a molecule to travel the free path.

(e) The molecules are perfectly elastic hard spheres. So, they do not exert any appreciable force of attraction or repulsion on one another or on the walls; except during the collisions.

From these assumptions we know that at any instant the molecules in a perfect gas can move along any direction with any velocity ranging from 0 to ∞ (We now know the limiting velocity is c, the velocity of light). The direction and magnitude of the velocity of a molecule are both distributed at random.

Let the velocity of a molecule be denoted by **v** having components p, q, r along x,y,z direction respectively ($p=v_x$, $q=v_y$, $r=v_z$)

$v^2=p^2+q^2+r^2$. v may vary from 0 to ∞, but p,q,r can vary from $-\infty$ to $+\infty$.

Though the Gas molecules are continuously colliding against one another, yet there is a steady state in which n_p the number of molecules with velocity component p is not affected by collisions. Otherwise, molecules would gather at place in larger number than at other. We wish to find out n_p in terms of p and other known quantities.

Let us proceed to calculate the pressure exerted by this gas.

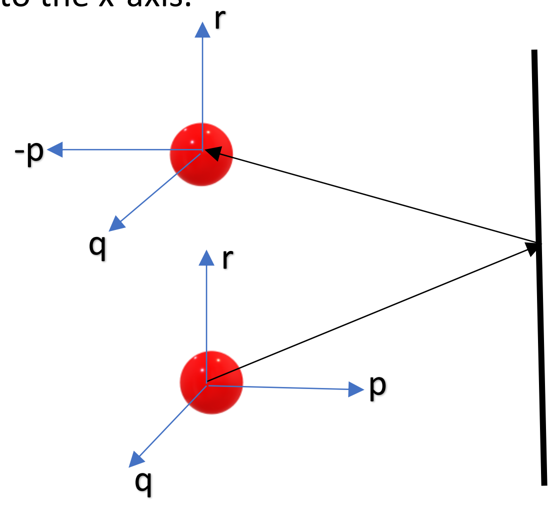
Two methods are there –

(a) Method of general dynamics
(b) Method of collisions

We will follow method (b)

Let us consider a perfect gas enclosed in a container and take the x-axis to be perpendicular to one surface of the container.

A molecule moving with x-component of velocity ‘p’ towards the wall collides with the latter. From the principle of conservation of energy and momentum we know that after the collision the x-component of velocity will be ‘-p’. Other components (y and z components) of velocity will not suffer a change, as the wall is perpendicular to the x-axis.



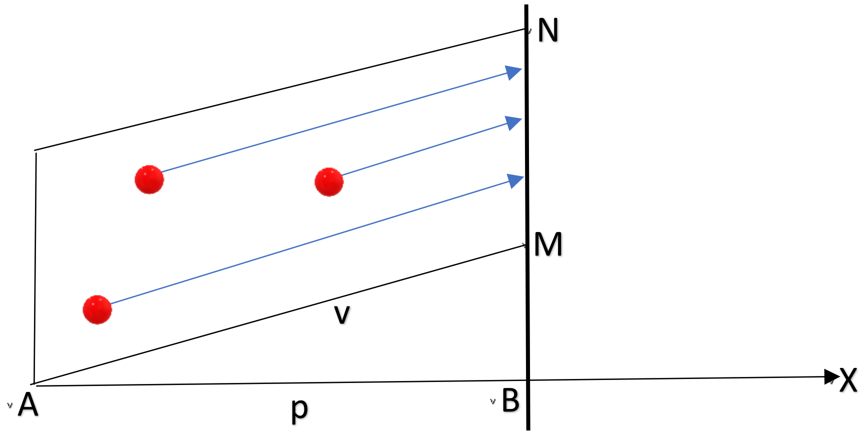
Hence the change in momentum suffered by that molecule during collision is 2mp [m is the mass of the gas molecule]. The other components of velocity do not suffer any change. So, they do not contribute to pressure.

Now pressure is equal to force per unit area.

Force is equal to the rate of change of momentum.

Therefore, pressure is equal to the change of momentum suffered by molecules striking unit area of the wall per second.

So, the pressure **P** = 2mp * number of impacts delivered on unit surface area of the wall.



Let us focus on a portion MN of wall.

Surface area of MN = δA

The molecules having x-component of velocity p, striking the area MN in time δt are contained inside the cylinder whose base area is MN, axis is AM=v δt and vertical height is AB=p δt. Any molecule outside this cylinder will not be able to deliver any impact on the surface area MN.

Volume of this cylinder is p δt δA.

Hence, the number of molecules having x-component of velocity p inside this cylinder is n_p p δt δA.

The total change of momentum experienced by these molecules is 2mp * n_p p δt δA.

Total change of momentum =

Force X time during which the change takes place =

Pressure X Area X time during which the change takes place

Hence, **P** δA δt = 2mp * n_p p δt δA

∴ **P** = $2mp^2 n_p$

Now pressure on the wall under consideration is caused by all the molecules having positive x-component of velocity.

[As the wall is perpendicular to the positive X-axis. Molecules with negative x-component of velocity will not strike on this wall]

$\therefore$ **P** = $2m \sum_{0}^{\infty} p^2 \, n_p$

Now let us try to calculate mean value of the quantity p^2 for all the molecules (denoted by $\overline{p^2}$).

n is the total number of molecules per c.c.

Considering all the molecules having x-component of velocity p we can write,

$$\overline{p^2} = \frac{\sum_{-\infty}^{+\infty} n_p p^2}{\sum_{-\infty}^{+\infty} n_p} = \frac{2\sum_{0}^{+\infty} n_p p^2}{n}$$

$\sum_{-\infty}^{+\infty} n_p$ = n, as we have calculated the sum for all values of p.

$$\therefore \sum_{0}^{\infty} p^2 \, n_p = \frac{1}{2} n \, \overline{p^2}$$

Here we have considered the wall to be perpendicular to the X-axis.

So, the pressure we have calculated here is the pressure along x-axis.

$\therefore P_x = mn\overline{p^2}$

Similarly, $P_y = mn\overline{q^2}$ and $P_z = mn\overline{r^2}$

We know that pressure in all directions should be same.

$P_x = P_y = P_z$

$\therefore mn\,\overline{p^2} = mn\,\overline{q^2} = mn\,\overline{r^2}$

$\therefore \overline{p^2} = \overline{q^2} = \overline{r^2}$

So, the mean square velocity in every direction is same. Otherwise, there would be accumulation of molecules in some part of the container.

Again, $\overline{v^2 = p^2 + q^2 + r^2}$; $\therefore p^2 = \frac{1}{3} v^2$

$\therefore P_x = P_y = P_z = P = \frac{1}{3} mnv^2$

$$\mathbf{P = \frac{1}{3} \rho v^2}$$ [ρ = density of gas]